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EJMA DESIGN EQUATIONS

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ABSTRACT

The Standards of the Expansion Joint Manufacturers Association (EJMA) are widely referenced for the design and application of round and rectangular metal expansion joints. The recent 6th edition contains numerous improvements designed to enhance usage. Changes and additions have been made to every section of the document. To appreciate the importance of these revisions an understanding of the background is important, especially for the bellows design equations. Consequently, this paper is written to provide a summary of the EJMA design equations with their theoretical bases and derivations as reflected in the 6th edition of the Standards.

BACKGROUND

The Expansion Joint Manf. Assoc. (EJMA) was founded in 1955 by a group of manufacturers who recognized the need for a consistent standard promoting the safe design and application of metal expansion joints. The first edition of the EJMA standards was published in 1958; however, it was not until the third edition, 1972 addenda, that equations for bellows design were introduced. These equations were based primarily on the work done by W.F. Anderson (1964). The shape factors (Cd, Cf, and Cp) which relate the behavior of a convolution to a simple strip beam were the most important contribution. The equations presented in the third edition thus provided the user with a means to verify the pressure capacity and fatigue life expectancy for a given bellows design. Later editions have expanded the equations to encompass more bellows types, provide greater accuracy, and furnish appropriate acceptance criteria.

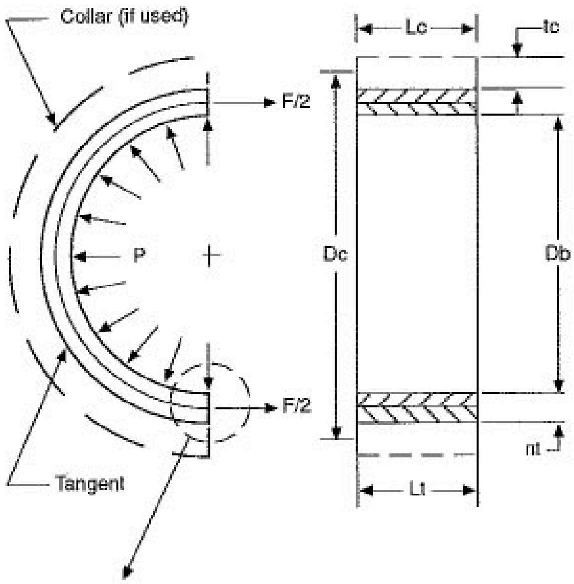
The 1993 sixth edition of the EJMA standards contains a total of thirty five (35) separate design equations with numerous improvements to enhance usage. With the information readily available, it may be tempting to use the standard as a "cookbook" for bellows design. Unfortunately, this approach could lead to a dangerous misapplication. Therefore, some theoretical understanding is important for proper usage of the latest equations.

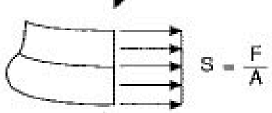
The following sections provide the theoretical basis and derivation for each of the design equations listed in the EJMA Standards, sixth edition (1993), for unreinforced bellows. Although not included in this paper, the design equations for the reinforced and toroidal type bellows have a similar formulation.

Bellows Tangent Circumferential Membrane Stress due to Pressure (S1)

The S1 Equation represents the circumferential membrane stress in the bellows tangent (cuff) due to pressure. It is a primary stress since strain does not relieve the applied load. To derive the equation, it is necessary to first create a free body diagram of the tangent with internal pressure (P) and length (Lt). Letting the sum of the forces equal zero for static equilibrium and solving for the force (F) gives eq. 1. The force (F) is applied to the cross sectional area (A). Without a collar, the area (A) is given by eq. 2. If a collar is used, the force (F) is applied to the total area given by eqs. 2 and 3. Equation 4 is the S1 equation taken directly from the EJMA standards. By manipulation of terms, the equation can be placed in the form of F/A which represents circumferential membrane stress as shown in eq. 5. Actually, the stress varies from a maximum value at the inside surface to a minimum value at the outside surface. For a better estimate of the max. tangent stress, Db+nt is substituted for Db in the numerator of eq. 5. The second quantity in the denominator of eq. 5 applies only when a collar is used. The terms Eb, Ec, and Dc apportion the stress between the tangent and the collar. The term k is a factor which considers the stiffening effect of the attachment weld and the end convolution on the tangent. It is based on shell theory (beam on elastic foundation) and empirical data.

The calculated value of S1 is defined as a primary membrane stress and must be less than the limit of Cwb times Sab. Considerations for a collar and for tangent weld joint efficiency (Cwb) were added to the S1 Equation in the sixth edition.





$$F = P\,D_b L_t \tag{1}$$

$$A\,(\text{tangent}) = 2ntL_t \tag{2}$$

$$A\,(\text{collar}) = 2\,t_c L_c \tag{3}$$

$$S_1 = \frac{P\,(D_b + nt)^2\;L_t\,E_b\,k}{2\,(nt\,E_b\,L_t\,(D_b + nt) + t_c\,k\,E_c L_c D_c)} \tag{4}$$

$$S_1 = \frac{\overbrace{P\,(D_b + nt)\;L_t}^{F}}{\underbrace{2\,nt\,L_t}_{A\,(\text{tangent})}\,\frac{1}{k} + \underbrace{2\,t_c L_c}_{A\,(\text{collar})\,(\text{if used})}\,\frac{D_c E_c}{(D_b + nt)E_b}} \tag{5}$$

Collar Circumferential Membrane Stress due to Pressure (S1')

The S1' equation represents the circumferential membrane stress in the collar due to pressure when a collar is used. The free body diagram and formulation is identical to that for the S1 equation. Equation 6 is the S1' Equation taken directly from the EJMA standards. By manipulation of terms, the equation can be placed in the form of F/A which represents membrane stress as shown in eq. 7. For a better estimate of the max. collar stress, Dc is substituted for Db in the numerator of equation 7. The terms Eb, Ec, and Db+nt apportion the stress between the tangent and the collar. The term k is a factor which considers the stiffening effect of the attachment weld and the end convolution on the tangent.

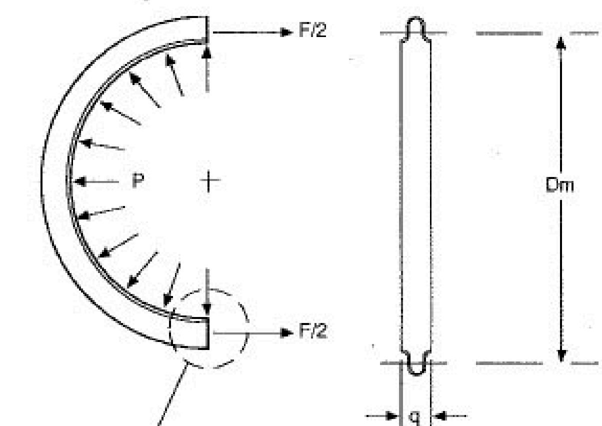
The calculated value of S1' is defined as a primary membrane stress and must be less than the limit of Cwc times Sac. The sixth edition added the consideration for collar length (Lc), collar diameter (Dc), collar weld joint efficiency (Cwc), and tangent stiffness factor (k) to the equation for S1'.

$$S_1' = \frac{P\,D_c^2\;L_t\;E_c\;k}{2\,(nt\,E_b L_t\,(D_b + nt) + t_c k\;E_c L_c D_c\,)} \tag{6}$$

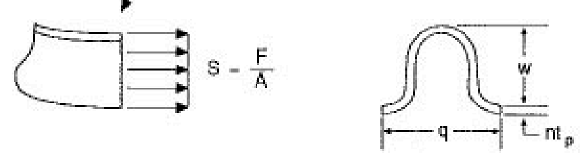
$$S_1' = \frac{\overbrace{P\,D_c\;L_t}^{F}}{\underbrace{2\,nt\;L_t}_{A\,(\text{tangent})}\,\frac{(D_b + nt)\,E_b}{D_c\,E_c k} + \underbrace{2\,t_c\,L_c}_{A\,(\text{collar})}} \tag{7}$$

Bellows Circumferential Membrane Stress due to Pressure (S2)

The S2 equation represents the circumferential membrane stress in the bellows convolutions due to pressure. It is a primary stress similar to S1. A free body diagram is created for one convolution with internal pressure (P) and length (q). Letting the sum of the forces equal zero and solving for the force gives eq. 8 where Dm is the mean diameter of the bellows. The force (F) is applied to the cross sectional area of the convolution (A) as given by eq. 9. Equation 10 is the S2 equation taken directly from the EJMA standards. By manipulation of terms, the equation can be placed in the form of F/A which represents circumferential membrane stress as shown in eq. 11. Therefore, the S2 equation reflects the circumferential membrane stress in the convolution.



$$F = P\,D_m\,q \tag{8}$$



$$A = 2\,nt_p\,(0.571\,q + 2w) \tag{9}$$

$$S_2 = \frac{P\,D_m}{2\,nt_p}\left(\frac{1}{0.571 + 2\,w/q}\right) \tag{10}$$

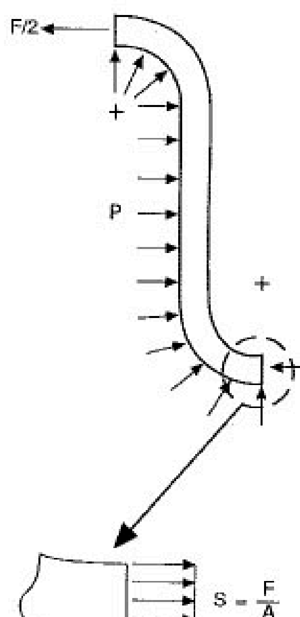
$$S_2 = \frac{\overbrace{P\;D_m q}^{F}}{\underbrace{2\;nt_p\;(0.571\;q\;+\;2\;w)}_{A}} \tag{11}$$

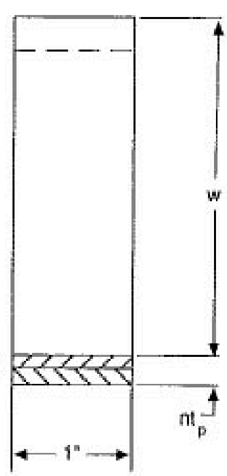
The calculated value for S2 is defined as a primary membrane stress and must be less than the limit of Cwb times Sab. Consideration for the weld joint efficiency (Cwb) was added in the sixth edition. Unpublished Finite Element Analysis (FEA) by the EJMA organization indicates that the actual S2 stress is not uniform around the convolution profile due to the influence of the S4 stress with Poisson's ratio. As expected, however, the algebraic mean of all the nodal stresses is very close to the value calculated by the S2 equation.

Bellows Meridional Membrane Stress due to Pressure (S3)

The S3 Equation represents the meridional membrane stress in the bellows convolutions due to pressure. It is a primary stress that follows the longitudinal axis of the bellows at the crest and root of the convolutions. A free body diagram is created for a one (1) inch wide strip of convolution with internal pressure (P). Letting the sum of the forces equal zero and solving for the force gives eq. 12 where w is the convolution height. The force (F) is applied to the total cross sectional area of the strip as given by eq. 13. Equation 14 is the equation for S3 taken directly from the EJMA standards. Since this Equation is in the form of F/A, the S3 equation represents meridional membrane stress in the convolution.

The calculated value of S3 is defined as a primary membrane stress and is combined with the S4 bending stress for comparison with the allowable limit. The S3 stress is normally less than 10 percent of the S4 value.





$$S = \frac{F}{A}$$

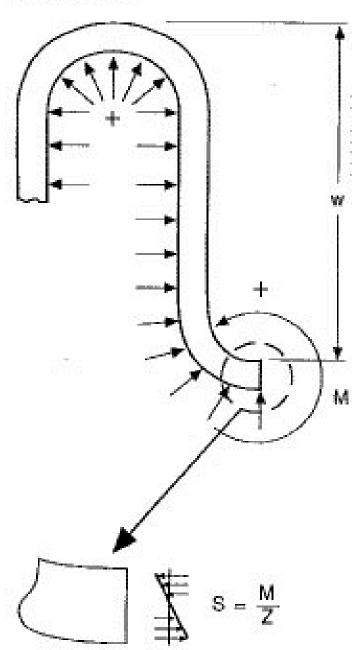
$$F = P\ w \qquad (12)$$

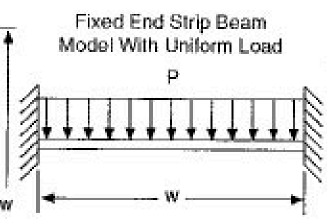
$$A = 2\ nt_p \qquad (13)$$

$$S_3 = \frac{\overbrace{P\ w}^{F}}{\underbrace{2\ nt_p}_{A}} \qquad (14)$$

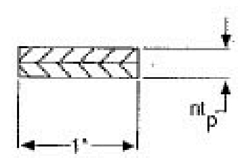
Bellows Meridional Bending Stress due to Pressure (S4)

The S4 equation represents the meridional bending stress in the bellows convolutions due to pressure. It is a primary stress that follows the longitudinal axis of the bellows across the convolutions. A free body diagram is created for a one (1) inch wide strip of convolution with internal pressure (P). The max. moment can be found by setting moments equal to zero. To find the value of the max. moment, the convolution is first modeled as a fixed end strip beam with a uniform pressure load P and length w. For the beam model, the max. moment (M) is given by eq. 15 where w is the convolution height. The moment (M) is applied to the section modulus of the cross section (Z) as given by eq. 16. Equation 17 is the equation for S4 taken directly from the EJMA standards. By manipulation of terms, the equation can be placed in the form of M/Z which represents meridional bending stress as shown in eq. 18. The term Cp is the shape factor developed by Anderson (1964) which relates the behavior of the bellows convolution segment to the simple strip beam model. Therefore, the S4 equation represents meridional bending stress in the convolution.





$$M = \frac{P\ w^2}{12}\ (\text{max.}) \qquad (15)$$



$$Z = \frac{nt_p^2}{6} \qquad (16)$$

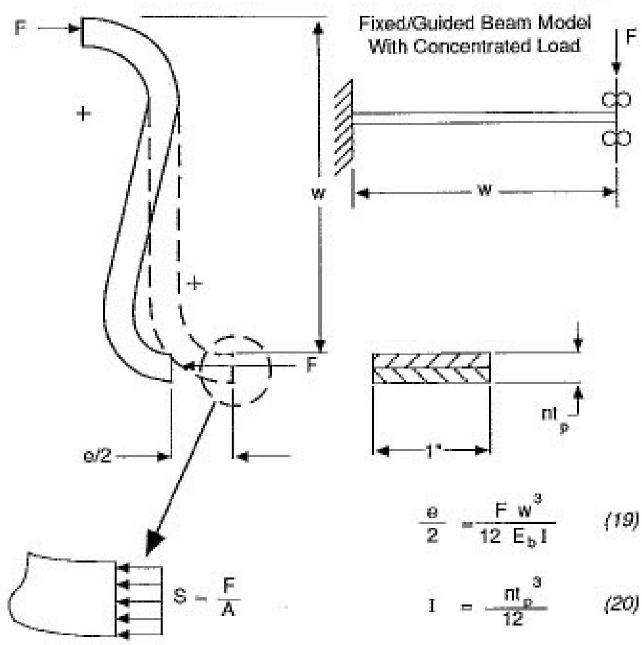
$$S_4 = \frac{P}{2n}\left(\frac{w}{t_p}\right)^2 C_p \qquad (17)$$

$$S_4 = \frac{Pw^2}{2nt_p^2}\ Cp = \frac{\overbrace{Pw^2/12}^{M}}{\underbrace{nt_p^2/6}_{Z}}\ C_p \qquad (18)$$

The calculated value of S4 is defined as a primary bending stress. As such, the sum of the primary membrane stress (S3) plus the primary bending stress (S4) must be less than the allowable limit. The limit is defined as the yield strength of the material in the as-installed condition or Cm times Sab. A wide ranging unpublished parametric study by the EJMA organization found that the average error between the calculated value of S3+S4 and FEA results was 2.8% with a maximum error of 8.2%.

Bellows Meridional Membrane Stress due to Deflection (S5)

The S5 equation represents the meridional membrane stress in the bellows convolutions due to deflection. It is a secondary stress since the applied load is limited by the deflection. It follows the longitudinal axis of the bellows. A free body diagram is created for a one (1) inch wide strip of convolution with deflection (e/2). The deflection generates a reaction force (F) which can be found by setting forces equal to zero. To find the value of the force (F), the convolution is first modeled as a fixed-guided strip beam with a concentrated load (F) and length (w). For the beam model, the deflection (e/2) is given by eq. 19. The moment of inertia (I) and cross sectional area (A) for the strip are given by eqs. 20 and 21, respectively. By manipulation of terms, the force (F) required to produce the deflection (e/2) is given by eq. 23. Since S5 is a membrane stress, the equation will have the form of F/A. By substitution and manipulation of terms, the equation for the membrane stress in the strip beam model is given by eq. 24. Finally, with the addition of the term Cf in the denominator, eq. 25 represents the S5 membrane stress. The term Cf is the shape factor developed by Anderson (1964) which relates the behavior of the bellows to the simple strip beam.



$$\frac{e}{2} = \frac{F\,w^3}{12\,E_b I} \qquad (19)$$

$$I = \frac{n t_p^3}{12} \qquad (20)$$

$$A = n t_p \qquad (21)$$

$$F = \frac{6\,E_b I\,e}{w^3} = \frac{6\,E_b n t_p^3 e}{12\,w^3} \qquad (23)$$

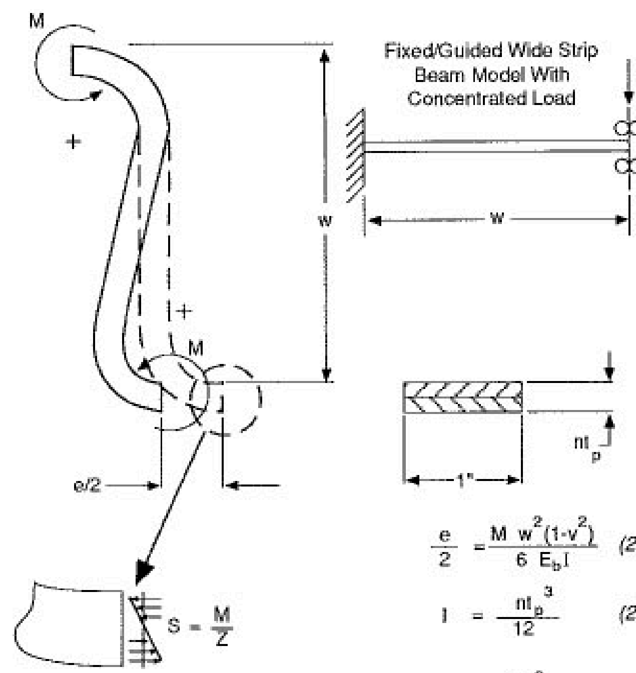
$$S = \frac{F}{A} = \frac{\frac{6E_b n t_p^3 e}{12\,w^3}}{n t_p} = \frac{6E_b n t_p^3 e}{12\,w^3\,n t_p} = \frac{E_b t_p^2 e}{2\,w^3} \qquad (24)$$

$$S_5 = \frac{E_b t_p^2 e}{2\,w^3 C_f} \qquad (25)$$

The calculated value of S5 is defined as a secondary membrane stress and is combined with the S6 bending stress for fatigue life analysis.

Bellows Meridional Bending Stress due to Deflection (S6)

The S6 equation represents the meridional bending stress in the bellows convolutions due to deflection. It is a secondary stress and follows the longitudinal axis of the bellows. Similar to the S5 formulation, a free body diagram is created for a one (1) inch wide strip of convolution with deflection (e/2). The deflection generates a reaction moment (M) which can be found by setting moments equal to zero. To find the value of the moment (M), the convolution is first modeled as a fixed-guided strip beam with a concentrated load (F) and a length (w). For the beam model, the deflection (e/2) is given by



$$\frac{e}{2} = \frac{M\,w^2(1-v^2)}{6\,E_b I} \qquad (26)$$

$$I = \frac{n t_p^3}{12} \qquad (27)$$

$$Z = \frac{n t_p^2}{6} \qquad (28)$$

$$M = \frac{3\,E_b I\,e}{w^2(1-v^2)} = \frac{3\,E_b n t_p^3 e}{12\,w^2(1-v^2)} \qquad (29)$$

$$S = \frac{M}{Z} = \frac{\frac{3E_b n t_p^3 e}{12\,w^2(1-v^2)}}{\frac{n t^3}{6}} = \frac{18E_b n t_p^3 e}{12\,w^2 n t^2(1-v^2)} = \frac{3E_b t_p e}{2\,w^2(1-0.3^2)}$$

$$S = \frac{5E_b t_p e}{3\,w^2} \qquad (30)$$

$$S_6 = \frac{5E_b t_p e}{3\,w^2 C_d} \qquad (31)$$

eq. 26 in terms of the max. moment. The term $(1-v^2)$ is included to consider the stiffening effect from a wide beam (Roark,1975). The moment of inertia (I) for the strip is given by eq. 27. The section modulus (Z) of the strip is given by eq. 28 and the value of Poisson's ratio (v) is taken as 0.3. By manipulation of terms, the max. moment (M) produced by the deflection (e/2) is given by eq. 29. Since S6 is a bending stress, the equation has the form of M/Z. By substitution and manipulation of terms, the equation for the max. bending stress in the strip beam model is given by eq. 30. Finally, with the addition of the term Cd in the denominator, eq. 31 represents the S6 bending stress. The term Cd is the shape factor developed by Anderson (1964) which relates the behavior of the bellows to the simple strip beam.

The calculated value of S6 is defined as a secondary bending stress. As such, the sum of the secondary membrane stress (S5) plus the secondary bending stress (S6) is used in the equation for bellows fatigue life (Nc). Unpublished parametric studies by the EJMA organization indicate that the calculated values of S5+S6 are within 10% of FEA results. Since the calculation is linear-elastic, the typical pseudo stress range for S5+S6 values is 200,000 to 300,000 psi.

Fatigue Life (Nc)

The Nc Equation as given by eq. 32 represents bellows fatigue life. It is the equation for a best fit curve that passes through a set of test data points plotted on a graph of total stress range (St) verses cycles to failure (Nc). The 300 series stainless steel test bellows were provided by several manufactures and cycle tested in the as-formed condition. The constants a, b, and c define the shape of the curve. The Ct term was included to adjust the curve for the effect of moderate temperatures on fatigue life but would not be applicable in the creep temperature range. Equation 33 represents the total stress range (St) and is a summation of meridional pressure and deflection stresses in the bellows.

$$Nc = \left(\frac{c}{S_t\,C_t - b}\right)^a \qquad (32)$$

$$St = 0.7\,(S_3 + S_4) + S_5 + S_6 \qquad (33)$$

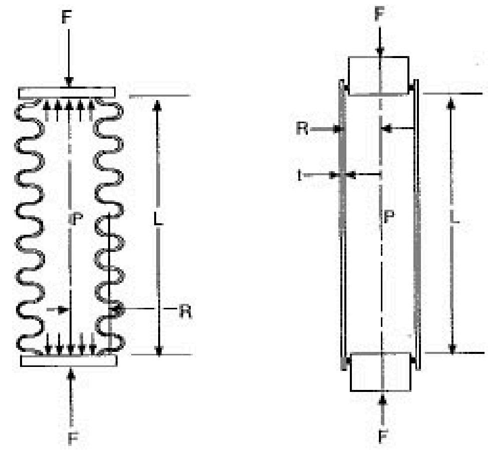
The pressure stresses S3 + S4 are included to account for the effect of concurrent pressure fluctuations on fatigue life. The multiplier of 0.7 is a factor which correlates the influence of the pressure stresses on fatigue test results. Comparative test data indicates that the fatigue life equation can be used to predict the average cycles to failure for other austenitic bellows materials of comparable yield strength. The sixth edition revised and relocated the temperature correction factor (Ct) in the equation to apply directly on total stress range.

Limiting Internal Design Pressure based on Column Instability (Psc)

The Psc equations represent the max. design pressure for a bellows based on column instability. Column instability is characterized by a gross lateral shift of the bellows centerline. Bellows become unstable and squirm when the lateral force which is proportional to internal pressure exceeds the restoring force due to bellows stiffness. In like manner, a structural column becomes unstable and buckles when the lateral force, which is proportional to the compressive load, exceeds the restoring force due to column stiffness. Testing has shown that bellows follow the classical pattern for slender structural columns under a compressive load. As the bellows length-over-diameter ratio (L/D) decreases, the critical instability pressure increases exponentially demonstrating elastic behavior. As the L/D ratio continues to decrease, an inflection point occurs where the critical instability pressure transitions to a maximum value at the length of one convolution which represents purely inelastic behavior. The distinct regions of the critical instability pressure curve are represented by the two (2) equations given in the EJMA standards.

To derive the equation for the elastic region of the instability curve, a bellows is assumed with ends capped, internal pressure (P), mean radius (R), and length (L). The bellows generates a longitudinal pressure thrust force (F) at each end. The same conditions can be duplicated by a fluid filled tube of length (L) and radius (R) with frictionless pistons in each end and internal pressure (P). The lateral movement of the column of fluid inside the tube is prevented only by the lateral stiffness of the tube and the column of fluid buckles only when the tube buckles. Therefore, if the tube has the same lateral stiffness as the bellows, instability would occur with the same internal pressure (P) and pressure thrust force (F). Note that the internal pressure (P) can be produced either by adding fluid or by application of the longitudinal force (F) to the end of the column of fluid. Using this principle, the fluid filled tube can be likened to a tubular column under a compressive load (F). From elastic column theory, buckling is a function only of length and lateral stiffness; therefore, the fluid filled tube can be analyzed for buckling as a traditional Euler column. Consequently, a tube model with the same length, radius, lateral stiffness, and compressive load (F) can be used to simulate the column instability of a bellows.

For the tube model, the cross sectional area (A) and moment of inertia (I) are given by eqs. 34 and 35, respectively. The critical load for the tube model (Pcr) is given by the Euler formula in eq. 36 which is set equal to the critical load (Fcr) generated by the bellows with pressure (Pcr) in eq. 37. Solving for the critical pressure (Pcr) gives eq. 38. The axial stiffness of the tube model (K) is given by eq. 39 which is set equal to the axial spring rate of the bellows in eq. 40. Solving for tE gives eq. 41 which is substituted into eq. 38 resulting in eq. 42 for critical pressure (Pcr) in the bellows. From a previous paper (Broyles,1989) empirical data was used to find an effective length coefficient (c) which correlates the equation to test results as given by eq. 43. Using a safety factor of 2.25, the limiting design pressure based upon elastic column instability (Psc) is given by eq. 44. The value of Psc must exceed the design pressure.



$$A = 2\pi R\ t \tag{34}$$

$$I = \pi R^3\ t \tag{35}$$

$$F_{cr} = \frac{\pi^2 E\ I}{(c\ L)^2} = \frac{\pi^3 R^3\ t\ E}{(c\ L)^2} \tag{36}$$

$$F_{cr} = P_{cr}\ \pi\ R^2\ \text{(Bellows)} = \frac{\pi^3 R^3 t\ E}{(c\ L)^2}\ \text{(Model)} \tag{37}$$

$$P_{cr} = \frac{\pi^3 R^3\ t\ E}{\pi\ R^2 (c\ L)^2} = \frac{\pi^2 R\ t\ E}{(c\ L)^2} \tag{38}$$

$$K = \frac{A\ E}{L} = \frac{2\ \pi\ R\ t\ E}{L} \tag{39}$$

$$K = \frac{f\,iu}{N}\ \text{(Bellows)} = \frac{2\pi\ R\ t\ E}{L}\ \text{(Model)} \tag{40}$$

$$t\,E = \frac{f\,iu\ L}{2\pi\ R\ N} \tag{41}$$

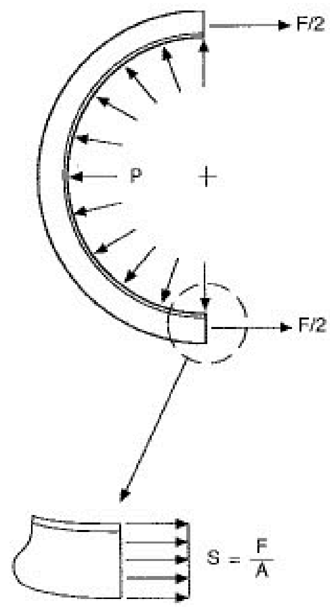
$$P_{cr} = \frac{\pi^2 R \left(\frac{f\,iu\ L}{2\pi\ R\ N}\right)}{(c\,L)^2} = \frac{\pi\ f\,iu}{2\ c^2 N\ L} = \frac{\pi\ f\,iu}{2\ c^2 N^2 q}\ \text{(Bellows)} \tag{42}$$

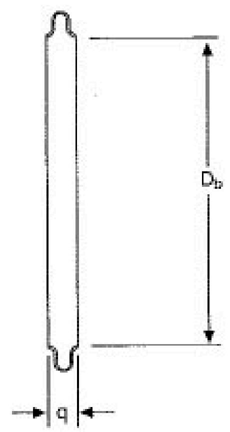
$$P_{cr} = \frac{\pi\ f\,iu}{2\,(0.809)^2\ N^2\ q} = \frac{0.764\,\pi\ f\,iu}{N^2\ q} \tag{43}$$

$$P_{sc} = \frac{P_{cr}}{2.25} = \frac{0.34\,\pi\ f\,iu}{N^2\ q} \tag{44}$$

To derive the equation for the inelastic region of the instability curve, it is necessary to find the pressure for the onset of fully inelastic behavior where the average circumferential stress equals the yield strength of the material. A free body diagram is created for the bellows with internal pressure (P) and length (q). Letting the sum of the forces equal zero and solving for the force is approximated by equation 45 where Db is the inside diameter of the bellows. The force (F) is applied to the cross sectional area of the convolution (Ac) as given by eq. 46 which represents the circumferential membrane stress in the convolution (S). By solving for the pressure (P) and substituting the yield strength (Sy) gives eq. 47 which represents the pressure for fully inelastic behavior. From a previous paper (Broyles, 1989), a correlation coefficient is applied to match test results in eq. 48. In addition, a reduction factor based on Lb/Db ratio is used to create the smooth transition from fully inelastic to elastic behavior at the inflection point of the instability curve. The transition point factor (Cz) locates the inflection point based upon empirical data from Broyles (1989). Using a safety factor of 2.25, the limiting design pressure based upon inelastic column instability (Psc) is given by eq. 49. The value with Psc must exceed the design pressure.

In summary, the two (2) eqs. 44 and 49 were added in the sixth edition to cover the complete range of bellows from low to high Lb/Db ratios and match actual test results for clearly defined column instability failures where both ends of the bellows are fixed. Just as with a structural column, the degree of fixity at each end of the bellows greatly affects the critical column squirm pressure.





$$F \cong P\ \ D_b q \tag{45}$$

$$S = \frac{F}{A} = \frac{P\ D_b q}{2\ A_c} \tag{46}$$

$$P = \frac{2\ A_c\ S}{D_b\ q} = \frac{2\ A_c\ S_y}{D_b\ q} \tag{47}$$

$$P_{cr} = \frac{2\ (0.653)\ A_c\ S_y}{D_b\ q}\left(1 - \frac{0.6\ L_b}{C_z\ D_b}\right) \tag{48}$$

$$P_{sc} = \frac{P_{cr}}{2.25} = \frac{0.58\ A_c\ S_y}{D_b\ q}\left(1 - \frac{0.6\ L_b}{C_z\ D_b}\right) \tag{49}$$

Limiting Internal Design Pressure based on Inplane Instability (Psi)

The Psi equation represents the max. design pressure for a bellows based on inplane instability. In the case of a beam, the localized buckling of the web and flange must be considered in addition to the overall buckling of the entire beam. This localized behavior in the beam is analogous to inplane instability in the bellows. It is characterized by a shift, rotation, or warping of one or more convolutions. Work by Becht (Becht, 1981) and testing by the EJMA organization (Thomas, 1984) indicates that inplane instability is associated with a high meridional bending stress (S4) and the yield strength of the material. It is not directly related to the Lb/Db ratio.

Equation 50 is the equation for S4 as taken directly from the EJMA standards. Solving for the pressure (P) gives eq. 51. From a previous paper (Broyles, 1989), testing indicates that even for very different convolution shapes inplane instability begins when the calculated value of S4 equals 1.575 Sy as given by eq. 52. Therefore, by substitution, the critical pressure is given by eq. 53. Using a safety factor of 2.25, the max. design pressure based upon inplane instability which was added in the sixth edition is given by eq. 54. The value of Psi must exceed the design pressure.

$$S_4 = \frac{P}{2_n}\left(\frac{w}{t_p}\right)^2 C_p \qquad (50)$$

$$P = \frac{2\ nt_p^2 S_4}{w^2 C_p} \qquad (51)$$

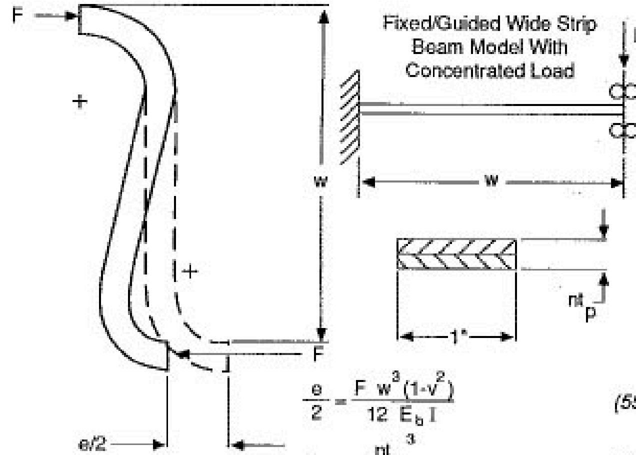
$$S_4 = 1.575\ S_y \qquad (52)$$

$$P_{cr} = \frac{2\ nt_p^2\,(1.575\ S_y)}{w^2 C_p} \qquad (53)$$

$$P_{si} = \frac{Pcr}{2.25} = \frac{1.4\ nt_p^2 S_y}{w^2\ C_p} \qquad (54)$$

Bellows Axial Elastic Spring Rate (fiμ)

The fiμ equation represents the bellows initial axial elastic spring rate and is the axial stiffness per convolution. Similar to the S5 formulation, a free body diagram is created for a one (1) inch wide strip of convolution with deflection (e/2). The deflection generates a reaction force (F) which can be found by setting forces equal to zero. To find the value of the force (F), the convolution is first modeled as a fixed-guided strip beam with a concentrated load (F) and a length (w). For the beam model, the deflection (e/2) is given by eq. 55. The term ($1-v^2$) is included to consider the stiffening effect from a wide beam (Roark, 1975). The moment of inertia (I) is given by eq. 56. By substitution and manipulation of terms, the force (F) required for a deflection (e/2) is given by eq. 57. Therefore, the spring rate for the strip beam model is given by eq. 58. Equation 59 gives the spring rate of the bellows where Dm is the mean diameter. By substitution and using a value of 0.3 for Poissons' ratio (v) gives the spring rate of the bellows based on the strip beam model. Finally, with the addition of the term Cf in the denominator, equation 61 represents the initial elastic spring rate in the bellows. The term Cf is the shape factor developed by Anderson (1964) which relates the behavior of the bellows to a simple strip beam.

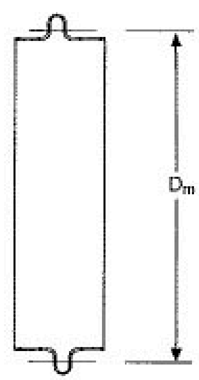


$$\frac{e}{2} = \frac{F\ w^3(1-v^2)}{12\ E_b I} \qquad (55)$$

$$I = \frac{nt_p^3}{12} \qquad (56)$$

$$F = \frac{6\ E_b\ I\ e}{w^3\,(1-v^2)} = \frac{6\ E_b\ nt_p^3 e}{12\ w^3\,(1-v^2)} \qquad (57)$$

$$\frac{F}{e} = \frac{6\ E_b\ nt_p^3}{12\ w^3(1-v^2)}\ (Model) \qquad (58)$$



$$f = \pi\ D_m \frac{F}{e}\ (Bellows) \qquad (59)$$

$$f = \frac{6\pi\ D_m E_b t_p^3 n}{12\ w^3(1-v^2)} = \frac{1.571\ D_m E_b\ t_p^3\ n}{w^3\,(1-0.3^2)} = \frac{1.7\ D_m\ E_b t_p^3 n}{w^3} \qquad (60)$$

$$f\,iu = \frac{1.7\ D_m\ E_b t_p^3 n}{w^3\ C_f} \qquad (61)$$

Pressure Testing

If the test pressure (Pt) is not greater than 1.5P, no additional calculations are required to confirm the bellows design. However, if the test pressure (Pt) exceeds 1.5 P, the calculations should be repeated at test pressure (Pt) and the results compared with the following acceptance criteria using room temperature material properties and room termperature allowable stress values for Sab and Sac:

S1 and S2 ≤ 1.5 Sab
S1' ≤ 1.5 Sac
S3 + S4 ≤ 1.5 Cm Sab
Psi and Psc ≥ Pt/1.5

The above evaluations will prevent permanent deformation and squirm of the bellows during the pressure test.

Reinforced Bellows Design Equations

The equations shown in the 6th edition for unreinforced and reinforced bellows are very similar. In fact, the equations for S1 and S1' are identical. If the terms Ar and Er are set equal to zero, the equations for S2 are also identical. The equation for S2' and S2" are derived by apportioning the stress between the bellows element, reinforcing ring, and fastener based upon their relative cross sectional areas and elastic modulii. For a reinforced bellows, the effective convolution height is given by w-Crq. The derivations for reinforced S3, S4, S5, S6, and fir are the same as unreinforced when w-Crq is substituted for w. The constant of 0.85 was included in the equations for S3 and S4 to correlate the calculated value with unpublished parametric FEA studies by the EJMA organization. The derivation of the reinforced fatigue life equation is the same as for unreinforced. The equation for reinforced column squirm (Psc) has the same derivation as for the elastic region of the unreinforced bellows column instability curve.

REFERENCES

Anderson, W. F., 1964, "Analysis of Stresses in Bellows," NAA-SR-4527, United States Atomic Energy Commission, Part 1 and 2.

Becht, C., 1981, "Root Bulge of Bellows," ASME PVP-Vol. 51, *Metallic Bellows and Expansion Joints*, pp. 105-117.

Broyles, R. K., 1989, "Bellows Instability," ASME PVP-Vol. 168, *Metallic Bellows and Expansion Joints - 1989*, pp. 37-43.

EJMA, 1993, *Standards of the Expansion Joint Manf. Assoc.*, 6th Edition, Tarrytown, N.Y.

Roark, R. J., 1975, *Formulas for Stress and Strain*, 5th Edition, McGraw Hill, New York, pp. 188-191.

Thomas, R. E., 1984, "Validation of Bellows Design Criteria by Testing," ASME PVP-Vol 83, *Metallic Bellows and Expansion Joints: Part II*, pp. 55-64.

APPENDIX - NOMENCLATURE

A_c = Cross sectional metal area of one bellows convolution (in.^{2}).

C_d, C_f, C_p = Factor used in specific design calculations to relate U-shaped bellows convolution segment behavior to a simple strip beam.

C_t = Temperature correction factor for bellows fatigue life below the creep range.

C_x = Transition point factor

D_b = Inside diameter of cylindrical tangent and bellows convolutions (in.).

D_c = Mean diameter of bellows tangent reinforcing collar (in.).

D_m = Mean diameter of bellows convolutions (in.).

E = Modulus of Elasticity at design temperature, unless otherwise specified, for material (psi.). Subscripts b and c, denote the bellows and reinforcement collar, respectively.

L_b = Bellows convoluted length (in.).

L_c = Bellows tangent collar length (in.).

L_t = Bellows tangent length (in.).

N = Number of convolutions in one bellows.

P = Design Pressure (psig.).

S_y = Yield strength at design temperature, unless otherwise specified, of bellows material in the as-formed (with cold work), or annealed (without cold work) conditions (psi.).

e = Total equivalent axial movement per convolution (in.).

$fi\mu$ = Unreinforced bellows theoretical initial axial elastic spring rate per convolution (lb./in. of movement per convolution).

k = A factor which considers the stiffening effect of the attachment weld and the end convolution on the pressure capacity of the bellows tangent.

n = Number of bellows material plies of thickness, t.

q = Convolution pitch, the distance between corresponding points of any two adjacent convolutions in a bellows (in.).

t = Bellows nominal material thickness of one ply (in.).

t_c = Bellows tangent reinforcing collar material thickness (in.).

t_p = Bellows material thickness for one ply, corrected for thinning during forming (in.).

w = Convolution height less the bellows material thickness, nt (in.).